Learning of Cyber-Physical Systems

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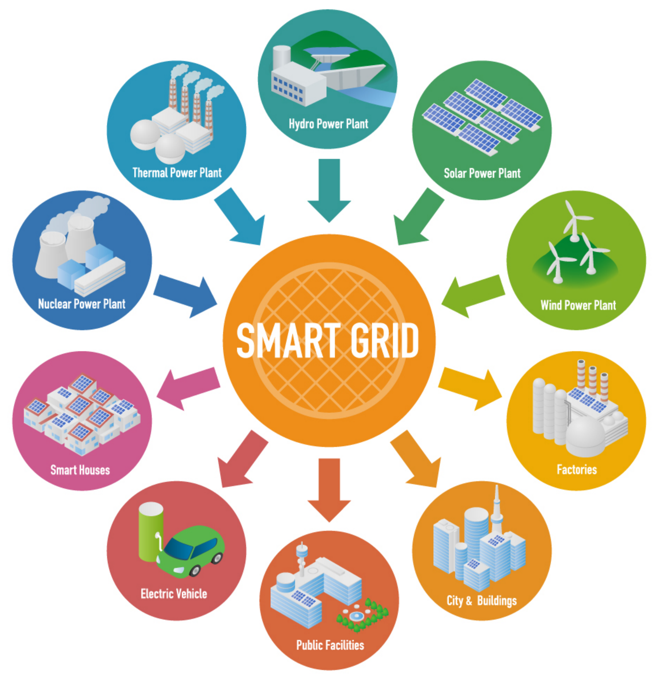
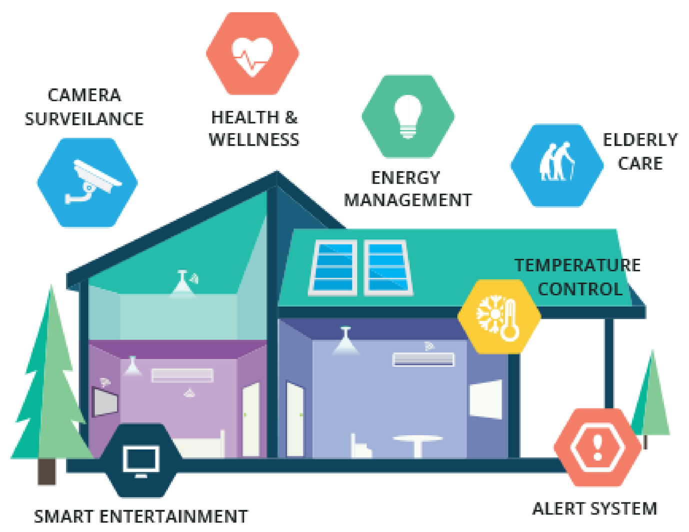
Introduction

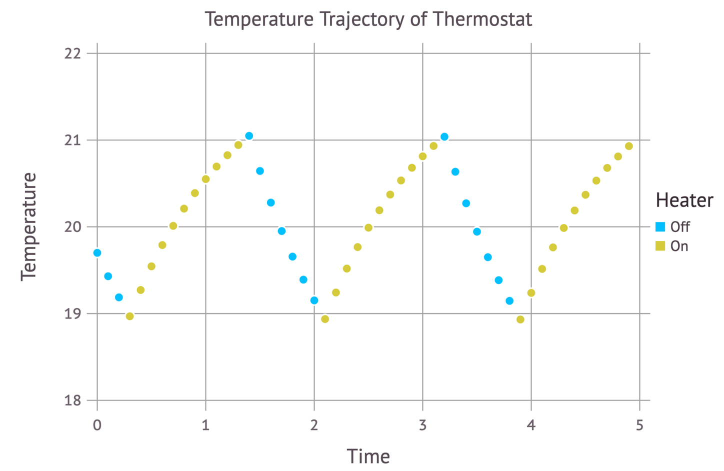
- *Cyber-physical systems* are embedded in physical environments and exert control via computation and devices.
  - Intelligent Hospitals and Smart Homes
  - Automated Highway Systems
  - Smart Grids and Renewable Energy

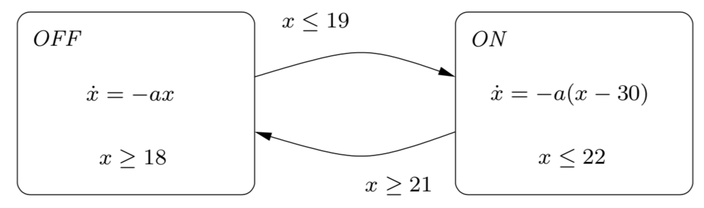




Smart Home Example

- Design goal: Maintain the room temperature within 19-21C.
- Data from Sensors; Control of Heaters
- Model: joint dynamics of physics and logic
- Interesting modeling questions:
  - Verification, Monitoring, Security, Optimization





Modeling

- *Hybrid automata* [1, 3]: model the joint dynamics of continuous, physical processes and discrete, control signals
  - Discrete states: $Q = \{q_1, q_2, \cdots\}$
  - Continuous states: $X \subseteq \mathbb{R}^n$
  - Vector field: $f : Q \times X \to X$
  - Edges: $E \subseteq Q \times Q$
  - Guard condition: $G : E \to 2^X$
- Sequence of intervals: $\tau = \{I_0, I_1, \cdots\}$
- Hybrid trajectory is $(\tau, q, x) = (\{I_i\}_0^N, \{q_i\}_0^N, \{x_i\}_0^N)$ where
  - Time interval: $I_i = [\tau_i, \tau_i']$
  - Discrete state function: $q_i : I_i \to Q$
  - Continuous state function: $x_i : I_i \to X$
- Execution:
  - Discrete dynamics: $\forall i : (q_i(\tau'), q_{i+1}(\tau)) \in E, x_i(\tau') \in G(q_i(\tau'), q_{i+1}(\tau))$
  - Continuous dynamics: $\forall i$
    - q_i is constant over I_i
    - x_i is the solution to $\dot{x}_i = f(q_i(t), x_i(t))$ over I_i

Learning

- How to learn an accurate model of the physical and logical components based only on data?
- Physical dynamics: learn the physical laws
- Logical dynamics: learn the finite automata
- Complete data likelihood:

$$\Pr(X, Q|\theta) = \Pr(X_0, Q_0|\theta) \prod_{t=1} \Pr(X_t|Q_t, X_{t-1}, \theta) \Pr(Q_t|Q_{t-1}, X_{t-1}, \theta)$$

- Useful for statistical inference on modeling questions.
- Extension beyond *Hidden Markov Chain* and *Hybrid Dynamical Systems*
- Complex relationships in physical states and combinatorial space of discrete states

Learning of physical processes

- Given time series of physical quantity $\{x\}^T$ and control signal $\{q\}^T$, estimate parameter $A_d = \begin{bmatrix} A_1 & \\ & A_2 \end{bmatrix}$.
- Assume physical processes are linear dynamical systems:

$$\dot{x} = A_d y + \epsilon \text{ where } y = \begin{bmatrix} x \\ q \end{bmatrix} \text{ and } \epsilon \sim \mathcal{N}(0, \sigma^2)$$

- This can be discretized using linear approximation:

$$x_{t+1} = Ay_t + \Delta t \epsilon_t \text{ where } A = \begin{bmatrix} \Delta t A_1 + I & \\ & A_2 \end{bmatrix}$$

- A can be estimated using *maximum likelihood estimation* [2]:

$$\widehat{A} = \Sigma_1 \Sigma_0^{-1}$$

where

$$\Sigma_0 = \frac{1}{T-1} \sum_t y_t y_t^T \text{ and } \Sigma_1 = \frac{1}{T-1} \sum_t y_{t+1} y_t^T$$

- Finally, $\widehat{A_d} = (\widehat{A} - I)/\Delta t$

Learning of discrete dynamics

- Given time series of physical quantity $\{x\}^T$ and control signal $\{q\}^T$, estimate distribution of next states:

$$\Pr(q_{t+1}|x_t, q_t, w) \text{ with parameter } w$$

- Assume logical dynamics are finite automata with linear constraints

$$w \cdot \begin{bmatrix} x \\ 1 \end{bmatrix} \geq 0$$

- Find the decision boundary in the space of x using *large-margin estimation*:

$$\min \|w\|_2^2$$

subject to

$$\forall t : \delta(q_{t+1}, q_t, x_t) \cdot \left(w \cdot \begin{bmatrix} x_t \\ 1 \end{bmatrix} \right) \geq 1$$

where

$$\delta(q_{t+1}, q_t, x_t) = \begin{cases} 1 & \text{if transits to } q_{t+1} \\ -1 & \text{else} \end{cases}$$

Next steps

- Evaluate on various real-world systems
- Consider relationships in continuous and discrete variables
- Partial observations, missing data and different forms of noise
- Formulate a learning method using mixed-variable graphical models
- Privacy issues

Conclusion

- Formulate the learning problem of cyber-physicl systems as learning the continuous and discrete dynamics
- Present the first solution using least-square maximum likelihood estimation and large margin estimation.
- Open source implementation in Julia: *https://github.com/zheguang/cyber-physical-learn*

References

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